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NON-NEWTONIAN LIQUID–AIR STRATIFIED FLOW THROUGH HORIZONTAL TUBES—II

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Abstract—Non-Newtonian liquid–gas stratified flow data were obtained using 0.052 and 0.025 m dia horizontal circular ducts. Unless the liquid velocity was very low, the flow pattern generally observed was non-uniform stratified flow having an interfacial level gradient between the two phases. The Heywood–Charles model is valid for predicting the pressure drop and liquid holdup in pseudoplastic (shear thinning) non-Newtonian liquid–gas uniform stratified flow. Two-phase drag reduction, which is predicted by the Heywood–Charles model did not occur because there was a transition to semi-slug flow before the model criteria were reached. Interfacial liquid and gas shear stresses were compared.

INTRODUCTION AND BACKGROUND

In the chemical process industries, non-Newtonian liquids, especially pseudoplastic (shear thinning) liquids, are encountered frequently. Several industrial applications utilize these liquid–gas mixtures flowing through horizontal tubes; therefore, there is a need to understand the hydrodynamics and transport behavior of non-Newtonian liquid–gas systems under adiabatic and diabatic flow conditions.

Stratified liquid–gas flow is a basic flow pattern from which other flow patterns develop as the liquid and gas flow rates are varied. In the previous paper Bishop & Deshpande (1986, this issue, pp. 957–975) point out that although conceptually simple, achieving stable uniform stratified flow using high-viscosity Newtonian liquids presents several difficulties not experienced with other liquid–gas flow patterns. No well-defined non-Newtonian liquid–gas stratified flow data for horizontal pipe flow were found in the literature. However, Mujawar & Rao (1981) suggest that some of their water-soluble polymer–air data were obtained in the stratified flow regime. Deshpande & Bishop (1983) reported preliminary information concerning stratified non-Newtonian liquid–gas flow through horizontal tubes.

Heywood & Charles (1979) published a non-Newtonian liquid–gas uniform stratified flow model to predict liquid holdup and two-phase pressure drop. In addition the model predicts the conditions under which the two-phase pressure drop is lower than for the non-Newtonian liquid flowing alone. This behavior is termed two-phase drag reduction and has been reported often for non-Newtonian liquid–air plug–slug flow (Chhabra *et al.* 1984; Mahalingam & Valle 1972). Two-phase drag reduction occurs only when the liquid phase is initially in laminar flow and should not be confused with single-phase drag reduction caused by adding small amounts of polymer to a turbulent flowing Newtonian liquid. The Heywood–Charles model, which has not been tested against data, is an extension of the Newtonian liquid–gas stratified flow model formulated by Taitel & Dukler (1976a). It is important to note that both models assume uniform stratified flow. The term uniform stratified flow means that there is no interfacial level gradient (ILG) and that the measured axial pressure gradient is equal in each phase. A basic assumption in the often-used Lockhart–Martinelli (1949) pressure drop model is that the flow is uniform and pressure gradients are equal in each phase.

The objectives of this paper are to: (1) report stratified non-Newtonian liquid–gas experimental holdup and pressure drop data; (2) compare these data with the predictions of the Heywood–Charles model; (3) assess two-phase drag reduction in stratified flow; and (4) contrast the behavior of uniform and non-uniform (ILG) stratified flow.

Many water-soluble polymer solutions tend to exhibit pseudoplastic shear thinning behavior. Moreover even in dilute solutions (0.5–1.5% by wt) the effective viscosities at typical stratified liquid velocities and single-phase shear rates are about 20 mPa s or higher. The stratified flow requirements of a low liquid flow velocity combined with the inherent high liquid viscosity generally places non-Newtonian liquid–gas stratified flow in the laminar liquid–laminar or turbulent gas flow regime.

Because non-Newtonian liquid-gas stratified flow data are not available, a review and analysis were made of published high-viscosity Newtonian liquid–gas stratified flow data collected during a period of over 35 years. Only three sources of data were found which reported holdup and pressure drop data for moderate viscosity (arbitrarily defined as being $\geqslant$ 5 mPa s) liquid–gas stratified flow mixtures (Hoogendoorn 1959; Jensen 1972; Agrawal 1971; Agrawal *et al.* 1973). Other parameters for these tests and the Mujawar–Rao (1981) tests are listed in table 1. The Hoogendoorn data are wavy and not smooth stratified flow. In all the experimental studies, the pressure drop was either measured in only one phase (Jensen 1972; Hoogendoorn 1959; Mujawar & Rao 1981) or measured using centerlines pressure taps (Agrawal 1971). Bishop & Deshpande (1986) found evidence of non-uniform ILG flow in the three sets of Newtonian liquid–gas data and recommended that the axial pressure gradient be measured in each phase in future experimental stratified flow studies using high-viscosity liquids.

Weisman *et al.* (1979) reported flow-pattern transition information for 75 and 150 mPa s liquids and Tochigin *et al.* (1983) gave information for liquid holdup and stratified–slug flow transition using 10–40 mPa s liquid–gas mixtures. Iwasyk & Green (1982) reported limited flow-pattern information using liquids having viscosities as high as 1.5–2.0 Pa s; however, no stratified flow data or discussion of this flow pattern was included in the paper.

MODEL DEVELOPMENT

General

Figure 1 illustrates two cases of stratified horizontal flow through circular tubes. Case I considers uniform flow where an interfacial level gradient can not be observed; Case II considers gradually varied non-uniform flow where the interfacial level gradient (ILG) is measurable. In both situations flow is steady and removed from entrance or exit influence where the flow might vary rapidly such as at a free overflow location.

The origin of ILG can be viewed as an attempt by the liquid phase to flow independently of the gas phase as in single-phase open-channel flow. In horizontal open-channel flow the

Table 1. High-viscosity liquid-gas stratified flow in horizontal tubes

Investigator	Tube diameter (m)	Tube length (m)	Type of combining tee used[a]	Phase ΔP measured	Gas velocity range (m/s)	Liquid velocity range (m/s)	Liquid viscosity range (mPa s)	System used
Agrawal (1971; Agrawal *et al.* 1973)	0.026	17.0	Simple tee	Centerline pressure Taps	0.11–6.1	0.014–0.061	5.0	Air–oil
Hoogendoorn[b] (1959)	0.14	8.0	Simple tee	Liquid	3.2–21.0	0.072	20.0	Air–oil
Jensen (1972)	0.0254 0.0381 0.0508	7.3	Simple tee	Gas	2.0–9.1	0.01–0.00075	55.0–310.0	Air–glycerol solution
Mujawar & Rao (1981)	0.0121	2.82	Simple tee	Liquid	0–4.5	0.33 and 0.54	4.0[c]	Air–sodium alginate solution

†[a]A simple tee is a device which combines the liquid as gas without any attempt to increase the degree of mixing.
[b]Stratified wavy data reported.
[c]Effective viscosity at 357 s^{-1} shear rate.

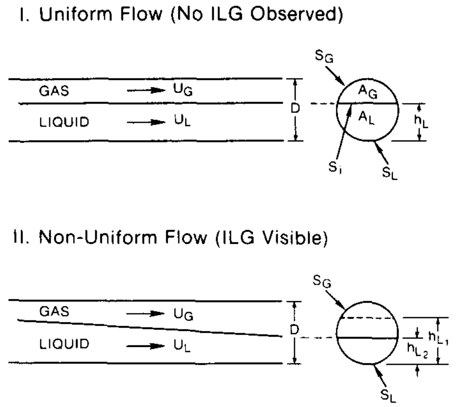

Figure 1. Parameters for stratified uniform and non-uniform (ILG) flow.

frictional resistance is balanced by a decrease in the depth of the liquid. During two-phase liquid–gas flow, an increase in the gas flow rate increases the liquid–gas interaction and partially suppresses ILG. Conversely, an increase in the liquid-phase frictional resistance acts to increase ILG. Tube length, tube diameter, entrance geometry and exit geometry probably affect ILG; however, the influence of these parameters has not been studied extensively.

The general one-dimensional steady-state mechanical energy balance equations, which include ILG, can be written for either non-Newtonian or Newtonian liquid–gas flow systems as:

$$-\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{\mathrm{TPL}} - g\,\rho_{\mathrm{L}}\frac{\mathrm{d}h_{\mathrm{L}}}{\mathrm{d}x} - \frac{\alpha\rho_{\mathrm{L}}}{2}\frac{\mathrm{d}(V_{\mathrm{L}}^2)}{\mathrm{d}x} = \frac{\tau_{\mathrm{WL}}S_{\mathrm{L}}}{A_{\mathrm{L}}} - \frac{\tau_{\mathrm{iL}}S_{\mathrm{i}}}{A_{\mathrm{L}}} \qquad [1]$$

and

$$-\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{\mathrm{TPG}} - \frac{\alpha\rho_{\mathrm{G}}}{2}\frac{\mathrm{d}(V_{\mathrm{G}}^2)}{\mathrm{d}x} = \frac{\tau_{\mathrm{WG}}S_{\mathrm{G}}}{A_{\mathrm{G}}} + \frac{\tau_{\mathrm{iG}}S_{\mathrm{i}}}{A_{\mathrm{G}}}. \qquad [2]$$

Here, $\mathrm{d}P/\mathrm{d}x$ is the pressure gradient, g is the acceleration due to gravity, ρ is the density, h is the height, V is the velocity, τ is the shear stress, S is the perimeter and A is the cross-sectional area for flow. The subscript TP stands for two-phase quantity, W for wall and i for interfacial conditions. In addition, subscripts L and G are used to indicate the liquid and gas phases, respectively. The kinetic energy correction term α for laminar flow of a non-Newtonian shear-thinning power-law model liquid flowing through a completely filled circular tube is

$$\alpha = \frac{(2n'+1)(5n'+3)}{3(3n'-1)^2}, \qquad [3]$$

where n' is the flow behavior index in the power-law rheological model. No data are available to define the kinetic energy correction factor for pseudoplastic fluid flowing through partially filled tubes. If the interface is horizontal, uniform stratified flow exists; the potential and kinetic energy terms are zero and [4] and [5] result:

$$-\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{\mathrm{TPL}} = \frac{\tau_{\mathrm{WL}}S_{\mathrm{L}}}{A_{\mathrm{L}}} - \frac{\tau_{\mathrm{iL}}S_{\mathrm{i}}}{A_{\mathrm{L}}} \qquad [4]$$

and

$$-\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{\mathrm{TPG}} = \frac{\tau_{\mathrm{WG}}S_{\mathrm{G}}}{A_{\mathrm{G}}} + \frac{\tau_{\mathrm{iG}}S_{\mathrm{i}}}{A_{\mathrm{G}}}. \qquad [5]$$

If it is also assumed that $\tau_{\mathrm{iL}} = \tau_{\mathrm{iG}}$, equality of the two-phase pressure gradient in each phase is also implied. Taitel & Dukler (1976a) combined [4] and [5] and obtained a dimensionless equation which can be used to predict liquid holdup and pressure drop in Newtonian liquid–gas uniform stratified flow:

$$\chi^2 F(m, q, R_L) - F\left(R_L, \frac{f_{iL}}{f_{WG}}, \frac{f_{iG}}{f_{WG}}\right) = 0. \qquad [6]$$

In [6] m and q are the exponents of the Reynolds number for the gas and liquid friction factors, respectively, R_L is the liquid holdup, χ^2 is the Lockhart–Martinelli (1949) parameter and F indicates a functional relationship. Equation [6] can be iterated to determine R_L if f_i/f_{WG} is known or assumed. Bishop & Deshpande (1986) concluded that the assumption of $f_i = f_{WG}$ is generally valid for uniform smooth stratified laminar Newtonian liquid–laminar or turbulent gas flow. Equation [1] and [2] can also be expressed in a similar dimensionless form:

$$\chi^2 F(m, q, R_L) - F\left(R_L, \frac{f_{iL}}{f_{WG}}, \frac{f_{iG}}{f_{WG}}\right) - Z = 0. \qquad [7]$$

The parameter Z contains the effect of the ILG-related terms. In non-uniform ILG stratified flow, the assumption of an equal measured axial pressure gradient in each phase is not valid:

$$\left(\frac{dP}{dx}\right)_{TPLM} = \left(\frac{dP}{dx}\right)_{TPL} + \rho_L g \frac{dh_L}{dx} \neq \left(\frac{dP}{dx}\right)_{TPGM}, \qquad [8]$$

where dh_L/dx is the liquid level gradient inside the tube and the subscripts TPLM and TPGM indicate the measured two-phase quantity for liquid and gas, respectively.

However the inequality between the measured gradients can be expressed as

$$\left(\frac{dP}{dx}\right)_{TPLM} = \Sigma^2 \left(\frac{dP}{dx}\right)_{TPGM}. \qquad [9]$$

Thus Σ^2 is a parameter which is equal to the ratio of the measured axial pressure gradient in each phase. If uniform stratified flow exists, $\Sigma^2 = 1$. Equations [4] and [5] can be combined using [9] and assuming negligible kinetic energy changes to give the dimensionless equation

$$-\chi^2 F(m, q, R_L, \Sigma^2) + F\left(R_L, \frac{f_{iL}}{f_{WG}}, \frac{f_{iG}}{f_{WG}}\right) = 0, \qquad [10]$$

which is similar to [6]. Thus the liquid holdup under non-uniform, stratified flow conditions is

$$R_L = F\left(\chi, m, q, \Sigma^2, \frac{f_{iL}}{f_{WG}}, \frac{f_{iG}}{f_{WG}}\right). \qquad [11]$$

If values of m, q, Σ^2, f_{iL}/f_{WG} and f_{iG}/f_{WG} are specified, [10] can be iterated to obtain values of the liquid holdup R_L. Simplification can be introduced by assuming, $f_{iL} = f_{iG}$ in non-uniform ILG stratified flow; however, this assumption can not be generally valid.

An explicit expression for the interfacial gradient in open-channel flow of a non-Newtonian liquid through a horizontal duct is

$$\frac{dh}{dx} = \frac{\dfrac{2K}{\rho g [D_H/2]^{n+1}} \left(\dfrac{3n+1}{n}\right)^n V_L^n + \sin\beta}{1 - Fr^2}, \qquad [12]$$

where the Froude number $Fr^2 = \alpha V_L^2 / g A_L \, dA_L/dh_L$, β is the inclination angle of the duct, D_H is the hydraulic diameter and K and n are consistency and flow behavior indices in the power-law model. Two features are embodied in [12]; conditions which reduce dh/dx to zero and criteria for sub- and supercritical flow behavior corresponding to that in open-channel flow. Critical flow exists if $Fr = 1$.

Non-Newtonian liquid–gas uniform stratified flow model

Heywood & Charles (1979) started with [4] and [5] and developed a model for uniform stratified flow where the liquid phase obeys a shear-thinning (pseudoplastic) or shear-thickening (dilatant) power-law model:

$$\tau = K\dot{\gamma}^n. \qquad [13]$$

In [13] K is the consistency index, $\dot{\gamma}$ is the shear rate and n is the flow behavior index. The Metzner–Reed form of the Reynolds number was also used in the development:

$$\mathrm{Re}_{\mathrm{MR}} = \frac{D_L^n V_L^{2-n}\rho_L}{K8^{n-1}\left(\dfrac{1+3n}{4n}\right)^n} \qquad [14]$$

where $\mathrm{Re}_{\mathrm{MR}}$ is the Metzner–Reed Reynolds number, D_L is the equivalent diameter and V_L is the *in situ* liquid velocity.

Equivalent diameters were defined as

$$D_L = \frac{4A_L}{S_L} \quad \text{and} \quad D_G = \frac{4A_G}{S_i + S_G}. \qquad [15]$$

Thus the liquid phase is assumed to flow in an open duct and the gas phase is assumed to flow in a closed duct. Implied in the gas-phase equivalent-diameter definition is equal shear stress at the interface and at the wall. Friction factor relations were

$$f_L = \frac{16}{\mathrm{Re}_{\mathrm{MR}}} \quad \text{and} \quad f_G = C_G\left(\frac{DV\rho}{\mu}\right)_G^{-m}, \qquad [16]$$

where f is the friction factor, C is a constant and μ is the viscosity.

Using the above liquid-phase friction factor implies that a shape factor is not required if the liquid, under laminar flow conditions, only partially fills the duct. Consistent with general usage:

$$C_G = 16 \quad \text{and} \quad m = 1, \text{ laminar gas flow;} \qquad [17]$$

and

$$C_G = 0.079 \quad \text{and} \quad m = 0.25, \text{ turbulent gas flow, smooth pipes.} \qquad [18]$$

The Fanning friction factors were defined by

$$\tau_{WL} = \left(\frac{f\rho V^2}{2}\right)_L \quad \text{and} \quad \tau_{WG} = \left(\frac{f\rho V^2}{2}\right)_G, \qquad [19]$$

$$\tau_i = \frac{f_i\rho_G(V_G - V_L)^2}{2} \simeq f_i\frac{\rho_G V_G^2}{2} \quad \text{if} \quad VG > V_L. \qquad [20]$$

Average velocities were used and isothermal conditions were assumed. The resulting equation for non-Newtonian liquid–gas uniform stratified flow, similar to [6], is

$$-\chi^2 F(m, n, R_L) + F\left(R_L, \frac{f_i}{f_{WG}}\right) = 0, \qquad [21]$$

also written as

$$\chi^2 = \frac{-\left(\dfrac{dP}{dx}\right)_{SL}}{-\left(\dfrac{dP}{dx}\right)_{SG}} = \frac{\bar{V}_G^{(2-m)}}{\bar{V}_L^n}\frac{\bar{D}_L^{1+n}}{4\bar{D}_G^m}\frac{1}{\bar{A}_G}\left(\bar{S}_G + \frac{\pi}{4}\frac{\bar{S}_i}{\bar{A}_L}\frac{f_i}{f_{WG}}\right), \qquad [22]$$

where all the parameters with over bars are functions of $\bar{h}_L = h_L/D$ and $\bar{D}_L = D_L/D$, $\bar{S}_L = S_L/D$, $\bar{A}_L = A_L/D^2$, $\bar{V}_L = V_L/V_{SL}$ [similar expressions are defined for the gas phase (Heywood & Charles 1979; Taitel & Dukler 1976a)].

Equation [21] states that for laminar liquid–turbulent gas uniform stratified flow and $f_i/f_{WG} = 1$:

$$\bar{h}_L \text{ or } R_L = F(\chi, n). \qquad [23]$$

The basis for the two-phase drag reduction prediction starts with the general expression for the Lockhart–Martinelli two-phase pressure drop parameter, ϕ_G^2 proposed by Taitel & Dukler (1976a):

$$\phi_G^2 = \frac{-\left(\frac{dP}{dx}\right)_{TP}}{-\left(\frac{dP}{dx}\right)_{SG}} = \frac{\bar{V}_G^2(\bar{D}_G\bar{V}_G)^{-m}}{4\bar{A}_G}\left(\bar{S}_G + \frac{f_i}{f_{WG}}\bar{S}_i\right). \qquad [24]$$

Using $f_i = f_{WG}$ and definition of ϕ_L^2 gives

$$\phi_L^2 = \frac{\bar{V}_G^{2-m}}{\chi^2\bar{D}_G^{1+m}}. \qquad [25]$$

Thus, for $\phi_L^2 < 1$ and for drag reduction to occur in stratified flow,

$$\bar{V}_G^{2-m} < \chi^2\bar{D}_G^{1+m}. \qquad [26]$$

If the holdup R_L is known, the above criterion can be used to determine whether drag reduction exists. In terms of the flow behavior index n,

$$\phi_L^2 = \frac{\bar{V}_L^n}{\bar{D}_L^{1+n}}\frac{(\bar{S}_G + \bar{S}_i)}{\left(\bar{S}_G + \frac{\pi}{4}\frac{\bar{S}_i}{\bar{A}_L}\right)}. \qquad [27]$$

It should be emphasized that uniform stratified flow was assumed throughout the model development. In non-uniform stratified flow, as discussed previously, $(dP/dx)_{TPL} \neq (dP/dx)_{TPG}$ and the parameters ϕ_L^2 and ϕ_G^2 are meaningless.

Typical predictions using the model are given for R_L and ϕ_L^2 at different values of n and χ in table 2. Observe that the model predicts $\phi_L^2 < 1$ in the range of $\chi > 3$ as n, the flow behavior index, increases from 0.35 to 1.0. Equations [1] and [2] can be reduced to give an expression similar to [22]:

$$\chi^2 = \frac{\bar{D}_L^n\bar{A}_L}{\bar{S}_L\bar{V}_L^n(\bar{D}_G\bar{V}_G)^m}\left\{\bar{V}_G^2\left[\frac{\bar{S}_G}{\bar{A}_G} + \left(\frac{f_{iL}}{f_{WG}}\frac{\bar{S}_i}{\bar{A}_L} + \frac{f_{iG}}{f_{WG}}\frac{\bar{S}_i}{\bar{A}_G}\right)\right] + \left[\frac{2D^{(1+m)}(\bar{D}_G\bar{V}_G)^m}{C_G\left(\frac{\rho_G}{\mu_G}\right)^{-m}\rho_G V_{SG}^{2-m}}\right]\right.$$
$$\left.\left[-g\rho_L\left(\frac{dh_L}{dx}\right) - \left(\frac{dP}{dx}\right)_{TPL} + \left(\frac{dP}{dx}\right)_{TPG} + \frac{\alpha\rho_G}{2}\frac{dV_G^2}{dx}\right]\right\}. \qquad [28]$$

It should be noted in [28] that the Lockhart–Martinelli parameter χ is no longer a unique function of n and R_L and the terms due to ILG are not known *a priori*. Interfacial level gradient can be measured during the experiments and can be used to determine the ILG terms in [28]. Equation [28] is an expanded form of [10] where the ILG terms were combined in the parameter Σ^2 and kinetic energy changes were assumed negligible.

Table 2. Predictions of liquid holdup and the two-phase pressure drop using the Heywood–Charles non-Newtonian liquid–gas uniform stratified flow model

	$n = 0.35$		$n = 0.7$		$n = 1.0$	
χ	R_L	ϕ_L^2	R_L	ϕ_L^2	R_L	ϕ_L^2
1	0.18	1.72	0.27	2.34	0.33	2.84
2	0.41	1.02	0.46	1.28	0.49	1.48
3	0.54	0.88	0.57	1.02	0.59	1.14
4	0.62	0.82	0.64	0.91	0.65	0.99
5	0.68	0.79	0.69	0.85	0.69	0.90

EXPERIMENTAL SETUP AND TEST PROCEDURE

The auxiliary loop, shown in figure 2, is part of the larger loop which was constructed to obtain heat transfer and hydrodynamic data. A longer test section could be placed in the auxiliary loop. Test sections were fabricated from 0.052 or 0.025 m i.d. transparent PVC pipe. The length between pressure taps was 3.5 m and the length between the liquid–air combining tee and the exit was 7.5 m. The test section unit was positioned horizontally using a surveyors transit. Several supports were used to minimize the vibrations. All internal recesses between the transparent pipe and the fittings were filled and sanded to provide a uniform unobstructed bore over the entire length of the test section unit. The average gas holdup in the test section was measured by using electrically operated valves and then backfilling the tube with the test solution. Holdup measurements were checked using a calibrated level indicator which was attached directly to the test section. The pressure drops in the liquid and in the gas phase were measured simultaneously during each test run. Inclined manometers measured the airphase pressure drop and U-tube manometers measured the liquid-phase pressure drop.

The liquid and the air flow rates were measured by a special electromagnetic flowmeter and rotameters, respectively. The electromagnetic flowmeter has a range of 5×10^{-5}–5×10^{-3} m^3/s (100:1). A digital indicator displayed the liquid flow rate. The low flow rate range of the meter was selected to provide the low liquid flow rates required for achieving stratified flow using high-viscosity liquids. The meter calibration was checked by weighing the liquid.

Air under pressure was filtered and then regulated. The air was metered by two rotameters which provided a 0.0077–0.30 standard m^3/min range.

The viscosity of the glycerol solutions was measured using a Fann V-G meter and also with a Brookfield viscometer; the measurements agreed very well for both the instruments. The single-phase friction factor data for glycerol solutions followed the usual laminar flow 16/Re fanning friction factor behavior. Air friction factor data followed the Blasius correlation in the turbulent flow regime.

Water-soluble polymer 7H4 SCMC was used. Each polymer solution was prepared in distilled and filtered water. The rheology of each polymer solution was established by using the test section as a pipeline viscometer. A check was made before and after each test run to determine the degree of polymer solution degradation; however, the polymer solutions were comparatively stable. The single-phase pressure drop data were used to construct rheograms and to obtain values of n, the flow behavior index and K, the flow consistency

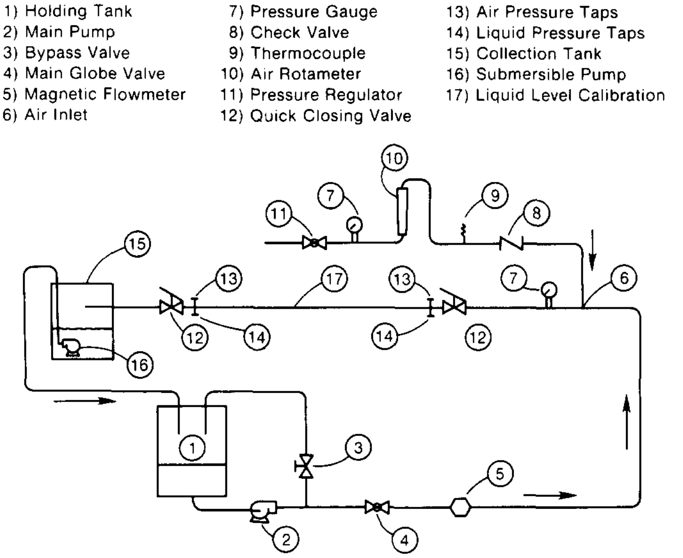

Figure 2. Schematic diagram of the experimental flow loop.

index. Temperature corrections to K were made where necessary. Corrections for entrance effects and kinetic energy losses were small. Because each rheogram was a straight line, a power-law model represented the pseudoplastic behavior over the desired shear rate–shear stress range. A sample rheogram for one of the 7H4 SCMC solutions is shown in figure 3. The rheological behavior of water-soluble polymers is such that the flow behavior index increases very little as temperature increases; however, a large decrease in the consistency factor occurs. Moreover for a power-law liquid:

$$n = n'; \quad K' = K\left(\frac{3n+1}{4n}\right)^n. \qquad [29]$$

Polymer solutions were prepared in a 0.3 m^3 capacity mixing tank. After being mixed, the solution was transferred to the main holding tank by a submersible pump from which the polymer solution was circulated through the test loop by a Moyno (0.265 m^3/min @ 8.274 MPa gauge pressure capacity) pump. Fine control on the solution flow rate was achieved by using the bypass valve and the main valve.

During each test, temperatures, flow rates, the pressure drop in each phase and the height of the liquid level interface were measured and recorded after sufficient time had been allowed to obtain steady flow. For a fixed superficial liquid flow rate, the gas flow rate was increased until stratified flow became unstable and a flow-pattern transition occurred.

Properties of the polymer solutions used and the range of parameters tested are given in table 3. It was difficult to obtain stable non-Newtonian liquid–gas stratified flow in the smaller diameter test section when high-viscosity liquids were used.

Because the liquid was in laminar flow, was non-Newtonian and because it only partially filled the test section during stratified flow, the question arose as to whether a shape factor was required in the friction factor relationship. For Newtonian liquid laminar flow through a cylindrical tube theory predicts $f\mathrm{Re}_L = 16$ if the pipe is flowing full and $f\mathrm{Re} = 15.5$ as $h_L/D \to 0$ (Shah & London 1978). However, the water data obtained by Straub *et al.* (1958) indicated that $f\mathrm{Re}_L = 16$ is valid over the entire range of h_L/D, where h_L is the liquid depth in the pipe and D is the pipe diameter. Test data were obtained for solutions with $n = 0.72$ and 0.79 over a range of $0.42 < h_L/D < 0.73$ and are shown in figure 4. The results justify the use of $f\mathrm{Re}_L = 16$ for this study. Kozicki & Tiu (1967) provided a theoretical discussion

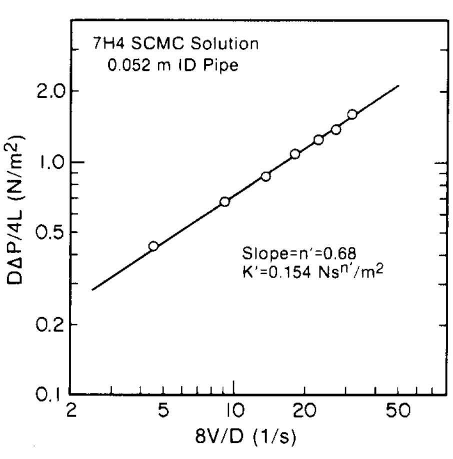

Figure 3. Shear stress–shear rate rheogram.

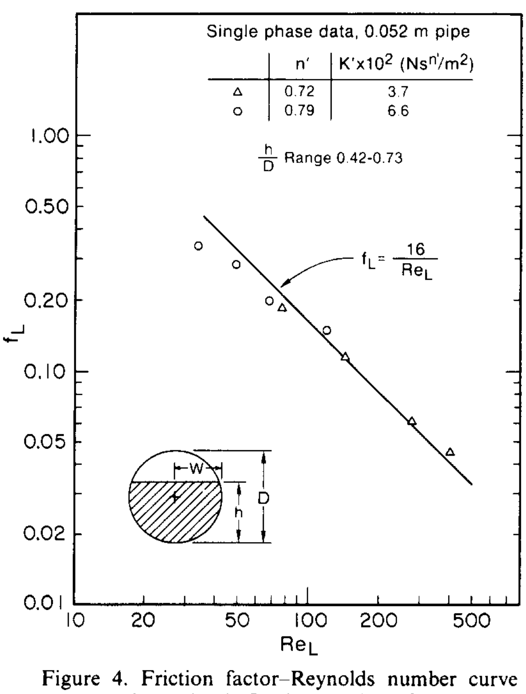

Figure 4. Friction factor–Reynolds number curve where pipe is flowing partially full.

Table 3. Physical properties of the test solutions

Solution	Temperature (K)	*Power-law Model Parameters* Flow behavior index, n'	Consistency index K' ($N s^{n'}/m^2$)	Density (kg/m^3)	Surface tension (mN/m)
Glycerol	300	1.00[a]	0.075[a]	1230	65.0
7H4 SCMC	304	0.68	0.15	1000	71.0
	305	0.72	0.14	1000	71.0
	305	0.80	0.08	1000	71.0
	309	0.85	0.03	1000	71.0
7H4 SCMC[b]	308	0.79	0.066	1000	—
	307	0.72	0.0375	1000	—
7H4 SCMC[c]	304	0.83	0.033	1000	—
	302	0.85	0.025	1000	—

[a]Brookfield viscometer.
[b]Single-phase open-chanel flow data collected in a 0.052 m dia pipe.
[c]Data collected in a 0.025 m dia pipe.

of the subject for non-Newtonian liquids. It can be determined from their equations that:

$$\frac{f_{MR}}{f^*} = \frac{Re^*}{Re_{MR}} = \left[\frac{\dfrac{3n+1}{4n}}{\dfrac{A+Bn}{n}}\right]^n, \qquad [30]$$

where f_{MR} denotes the friction factor obtained using the Metzner–Reed Reynolds number and f^* is friction factor defined by Kozicki & Tiu (1967). The shape factors A and B used in the definition of Re* change very little from the values of $A = 0.25$ and $B = 0.75$ for $h/W = 1.0$ as h/W changes from 1.0 to 0.125 (Tiu & Kozicki 1969); W is half the width of the liquid–gas interface in the tube. Therefore, from [30] it can be shown that for various flow behavior indices deviation from $f = 16/Re$ is very small up to $h/W = 0.125$. For example, for $n = 0.5$ the ratio of f_{MR}/f^* varies from unity for $h/W = 1.0$ to a value of 1.042 for $h/W = 0.25$.

The analysis in this paper assumes no effective slip of the non-Newtonian liquid at the wall. If this assumption is valid, a log–log plot of $Q/\pi r^3 \tau_W$ vs τ_W should produce a single straight line for different pipe diameters (Skelland 1967). The results obtained in this study using 0.025 and 0.052 m dia tubes are given in figure 5 and validate the "no-effective slip" assumption.

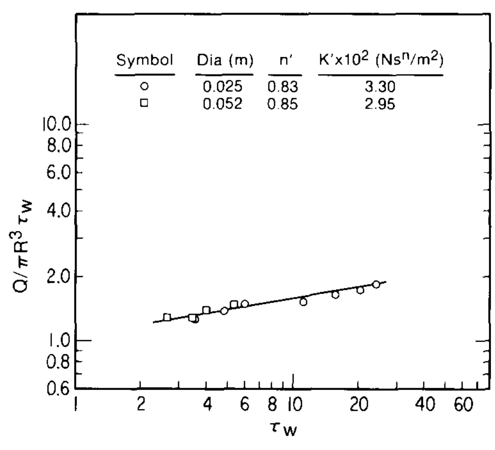

Figure 3. Plot of $Q/\pi R^3 \tau_W$ vs τ_W for SCMC solution to check for effective slip at wall.

RESULTS AND DISCUSSION

Visual observations of flow patterns were made during this study. Pictures of smooth and wavy stratified flow are shown in figure 6. There was virtually no stable wavy flow pattern for liquids having high effective viscosities (defined as the viscosity which makes the Poiseuille equation agree with the experimental single-phase-flow pressure drop). Moreover, an abrupt transition generally occurred from what appeared to be smooth stratified flow to a semi-slug-type flow as the air velocity was increased at a fixed liquid velocity [Sakaguchi *et al.* (1979) called attention to semi-slug flow]. Jensen (1972) reported a similar abrupt transition from stratified flow if the Newtonian liquids had a viscoscity > 240 mPa s. An explanation for this behavior is that the SS–WS (stratified smooth to stratified wavy) flow-pattern transition boundary and the WS–A (wavy stratified to annular) [or the SS–I (smooth stratified to intermittent)] flow-pattern transition boundary approach each other, thereby reducing the wavy stratified flow region as the liquid viscosity is increased. The Taitel–Dukler (1976b) flow-pattern map predicts that the SS–WS transition boundary is proportional to the square root of liquid viscosity. The viscosity dependency for the SS–I and WS–A boundary is not explicit and is expected to be weaker than that for the SS–WS boundary. Therefore, as viscosity increases there is a greater change in the SS–WS boundary which reduces the wavy stratified flow region. This behavior is not predicted by the Weisman *et al.* (1979) flow-pattern map because the SS–WS transition is indicated to be independent of liquid viscosity. The Baker (1954) and the Mandhane *et al.* (1974) flow-pattern maps have viscosity in the map coordinates, however, the dependency is only $\mu^{0.33}$ and $\mu^{0.2}$, respectively, and the high-viscosity data used are not identified clearly by Mandhane *et al.* Unfortunately, over the years, most liquid–gas studies (and flow-pattern maps derived from these studies) were made using only water–air mixtures. Moderate-viscosity liquids (viscosity > 5 mPa s) have not been studied adequately.

Sakaguchi *et al.* (1979) concluded that the developing distances for flow patterns other than annular flow are very short. Using different tube length-to-diameter ratios they also demonstrated that the largest effect of test section length occurred in the transition from wavy stratified to intermittent flow. This transition took place at lower liquid and gas velocites as the test section length was increased.

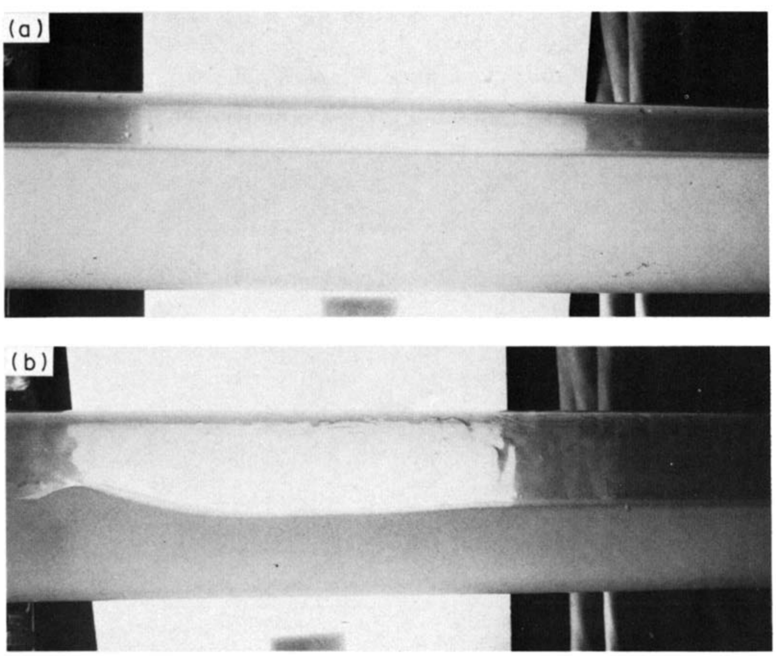

Figure 6. Photographs of flow patterns in a 0.052 m dia tube ($V_{SL} = 0.015$ m/s).

In the current study, two important observations were made.

(1) Most of the stratified flow data obtained was in the non-uniform region. This fact was determined simply by measuring the pressure gradient in each phase and observing that $(\Delta P/\Delta L)_{TPLM} \neq (\Delta P/\Delta L)_{TPGM}$.

(2) The range of polymer solution velocities over which smooth uniform stratified flow could be obtained was considerably below the typical value of 0.2 m/s proposed for water–air mixtures by Weisman *et al.* (1979), Taitel & Dukler (1976b) and Mandhane *et al.* (1974). Moreover, when the results of Weisman *et al.* (1979) for 75 and 150 mPa s liquids are compared with the air–water results they suggest that the stratified–slug flow transition occurs at a liquid superficial velocity of about 0.2 m/s, essentially independent of the liquid viscosity. In the current study, the range of stratified flow was restricted to lower liquid velocities as the liquid viscosity increased. A similar trend was found in the data of Jensen (1972) who had to use a liquid velocity in the range of 7.45×10^{-4}–4×10^{-3} m/s in a 0.038 m dia pipe to achieve uniform stratified flow where the liquid viscosity was 310 mPa s.

Bishop & Deshpande (1986) introduced the parameter Σ^2 to indicate the magnitude of non-uniformity in stratified flow where,

$$\Sigma^2 = \frac{\left(\frac{\Delta P}{\Delta L}\right)_{TPLM}}{\left(\frac{\Delta P}{\Delta L}\right)_{TPGM}}. \qquad [31]$$

If $\Sigma^2 = 1$, the interfacial level is horizontal, the measured two-phase pressure gradients are equal in the liquid and the gas phase, no ILG exists and hence the stratified flow is uniform. The measured value of the liquid-phase pressure gradient includes the potential head pressure gradient, or ILG, which is the cause of non-uniform stratified flow.

Data from a typical series of test runs are tabulated in table 4. Even at a low (0.015 m/s) superficial liquid velocity a value of V_G (the *in situ* gas velocity) of approx. 4.5 m/s was required to achieve uniform flow, i.e. $\Sigma^2 = 1$. Observe that the magnitude of Σ decreases as the gas velocity increases until a constant value of $\Sigma = 1$ is reached; at this condition the interface is horizontal and uniform stratified flow exists. The magnitude of Σ (or Σ^2) is not known *a priori* for a given set of conditions. The Lockhart–Martinelli parameters of ϕ_L^2 and ϕ_G^2 have no meaning if $\Sigma^2 > 1$ because the axial pressure gradient is not equal in the liquid and gas phases. The data show that the transition from stratified to semi-slug flow occurred at a superficial gas velocity $V_{SG} > 4.38$ m/s ($V_G > 6.0$).

In figure 7 the square root of the stratified flow parameter Σ^2 is plotted (for turbulent gas flow only) vs χ, the square root of the Lockhart–Martinelli parameter, for four different

Table 4. Stratified flow data

V_G (m/s)	Re_L	$Re_G \times 10^{-3}$	χ	R_L	ϕ_L^2	Σ^2
1.1	75	2.35	5.9	0.42	a	6.0
2.0	78	4.54	2.7	0.41	a	3.0
3.0	77	6.75	1.9	0.42	a	1.7
3.8	81	8.80	1.5	0.39	a	1.4
4.3	88	10.5	1.3	0.35	2.1	1.1
5.0	91	12.3	1.1	0.33	2.3	1.0
5.7	95	14.4	0.93	0.31	2.5	1.0
6.0	103	15.5	0.90	0.27	2.9	1.0
>6.0	Transition from stratified to semi-slug flow occurred					

[a] ϕ_L^2 Has no meaning if $\Sigma^2 > 1.0$; this indicates non-uniform ILG flow.
Polymer solution: 7H4 SCMC
$n' = 0.85$
$K' = 0.03$ N s$^{n'}$/m^2
$V_{SL} = 0.015$ m/s
$\mu_L = 20$–22 mPa s

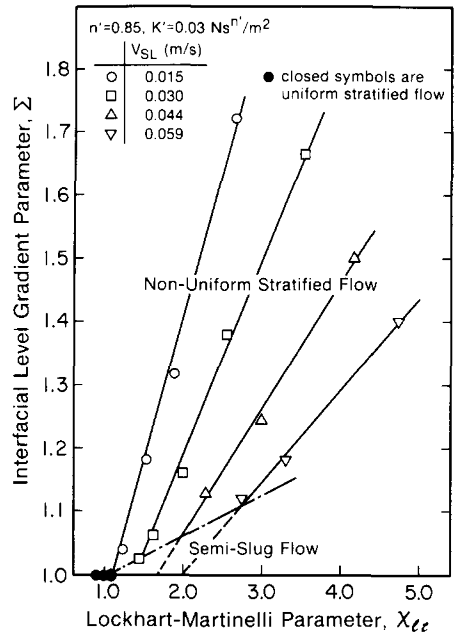

Figure 7. ILG parameter (Σ) vs Lockhart–Martinelli parameter χ for different liquid velocities.

superficial polymer solution velocities (0.015, 0.030, 0.044 and 0.059 m/s) using the same polymer solution (7H4 SCMC) noted in table 3. At a given value of χ it is seen that although the magnitude of Σ increases as the superficial liquid velocity decreases; the slopes of the straight lines decrease as the liquid velocity increases. Thus with an increase in gas velocity ILG decreases and the non-uniform stratified flow is transformed into the uniform stratified condition, as perviously discussed. Thus, for a specified value of χ, the value of Σ is lower for a higher liquid superficial velocity because a higher gas flow rate is required to obtain the same value of χ at the higher liquid flow rate; moreover, the gas–liquid interaction is greater at higher liquid and gas flow rates. Only for the lowest liquid velocity tested could uniform stratified flow be obtained ($\Sigma = 1$ reached). For the other three liquid velocities tested, a semi-slug-type transition took place before $\Sigma = 1$ was achieved. The trend also shows that as V_{SL} decreases from 0.015 m/s the region of uniform stratified flow increases. Inspection of figure 7 also suggests that a necessary condition for uniform stratified flow to exist is $\chi \approx 1$. This criterion is also supported by the Newtonian high liquid viscosity data of Jensen (1972). Some measure of satisfaction would result if $\chi \approx 1$ could be shown to be both a necessary and sufficient condtion for achieving uniform stratified flow because in that case the effects of other parameters on non-uniform ILG stratified flow, such as tube length, entrance and exit conditions and liquid–gas combinations used, would be minimized. The liquid flow rate corresponding to a superficial velocity of 0.015 m/s in a 0.052 m dia pipe was below the manufacturer's calibrated range of the electromagnetic flowmeter. Hence, experimental data for a lower liquid flow velocity were not collected. Moreover, the objective of determining the limiting liquid velocity required to reach uniform stratified flow had been accomplished. In summary, for a given geometry

$$\Sigma^2 = F(V_{SL}, \chi, n', K') \qquad [32]$$

in pseudoplastic non-Newtonian liquid–gas stratified flow. The corresponding functional relationship for Newtonian liquid–gas stratified flow would be

$$\Sigma^2 = F(V_{SL}, \chi). \qquad [33]$$

Unfortunately Σ^2 was not measured in stratified flow experiments performed by previous investigators who used moderate- or high-viscosity Newtonian liquids.

Another objective of this study was to determine whether two-phase drag reduction occurs in non-Newtonian liquid–gas stratified flow, as predicted by the Heywood–Charles

(1979) model. From table 2, drag reduction (i.e. $\phi_L^2 < 1$) would be predicted for $n' = 0.85$ if $\chi > 4.0$. Figure 7 shows that only non-uniform stratified flow would exist or a transition to semi-slug flow would occur at $\chi = 4$ if $V_{SL} > 0.015$ m/s. Therefore, even if a transition from non-uniform stratified flow to semi-slug flow does not occur at $\chi = 4$, the concept of ϕ_L^2 and ϕ_G^2 is not valid because there is no common basis for their calculation, i.e. $(\Delta P/\Delta L)_{TPL} \neq (\Delta P/\Delta L)_{TPG}$ or $\Sigma^2 \neq 1$. In contrast to the often reported two-phase drag reduction in plug–slug non-Newtonian liquid–gas mixtures (Chhabra *et al.* 1984; Mahalingam & Valle 1972) the results of the current study suggest that two-phase drag reduction can not be achieved in stratified flow of non-Newtonian liquid–gas mixtures. It is interesting that two-phase drag reduction has been observed experimentally in non-Newtonian liquid–gas plug–slug flow and in Newtonian liquid–liquid plug–slug and concentric flow patterns (Charles *et al.* 1961). But no definite experimental drag-reduction data have been reported for stratified flow for either the liquid–liquid or liquid–gas system. If two-phase drag reduction is restricted to those situations where streamline flow patterns exist at the head of an elongated bubble then drag reduction could not exist in stratified liquid–gas or liquid–liquid flow; Oliver & Young Hoon (1968) discuss the effect of recirculating and streamline flow patterns in drag reduction and heat transfer.

These results confirm the importance of measuring the pressure gradient in each phase in any stratified flow experiment and particularly where high-viscosity Newtonian liquids or polymer solutions are used. Stratified flow data are inconclusive without these measurements (for more basic studies, instrumentation should also be provided to measure the liquid height at various locations in the test section). If the pressure gradient is measured only in the gas phase, as is often done, two-phase drag reduction could be indicated for a number of the test data because the measured pressure gradient in the gas phase is always less than that in the liquid phase in non-uniform ILG stratified flow. Such a situation might have existed in the stratified flow data of Agrawal *et al.* (1973), where values of $\phi_L^2 < 1$ can be calculated using pressure drop data obtained using centerline pressure taps.

Figure 8 is a plot of Σ Vs χ, where all data are at the same superficial liquid velocity (0.015 m/s) but have different flow behavior and consistency indices. It can be seen that uniform stratified flow could be obtained only for the least viscous and the least non-Newtonian solution at high gas velocities. For other solutions, transition to semi-slug-type flow occurred before ILG could be eliminated completely. As n, the flow behavior index decreases, the slopes of the lines also decrease irrespective of the higher consistency of the solution at lower n. This indicates less interaction for the most pseudoplastic fluid. At the same χ, the values of Σ are higher for the less viscous solution because lower gas flow rates are required to get the same χ as liquid viscosity decreases.

Glycerol data were collected initially in a setup where the total length of pipe was not straight. The values of Σ were higher for glycerol solution compared to those for polymer solutions. The rate of change of Σ with χ was the same as that for a polymer having the highest value of the flow index. The rate of change of liquid holdup with gas velocity for glycerol was higher when compared with that of the polymer solution of similar effective viscosity. These results indicate higher gas–liquid interaction for a Newtonian solution. However, the importance of differences in geometries was not assessed.

Figure 9 shows that in non-uniform stratified flow there is a liquid holdup dependency on the liquid velocity and that over a wide range of χ values, the holdup is more insensitive to changes in χ than would be predicted by uniform stratified flow equations. This behavior illustrates the restricted interfacial liquid–gas interaction and the ineffectiveness of the gas-phase shearing stress. Insensitivity of holdup to decreasing χ is seen also in the Gazley (1948) non-uniform stratified flow data for an air–water system. The general behavior illustrated is characteristic of both non-uniform ILG Newtonion and non-Newtonian liquid–gas stratified flow.

At lower values of χ the holdup data agree very well with the predicted values for uniform stratified flow where liquid holdup is independent of the liquid flow rate. At a fixed value of χ, and equal liquid velocities, the higher holdup values obtained in the polymer

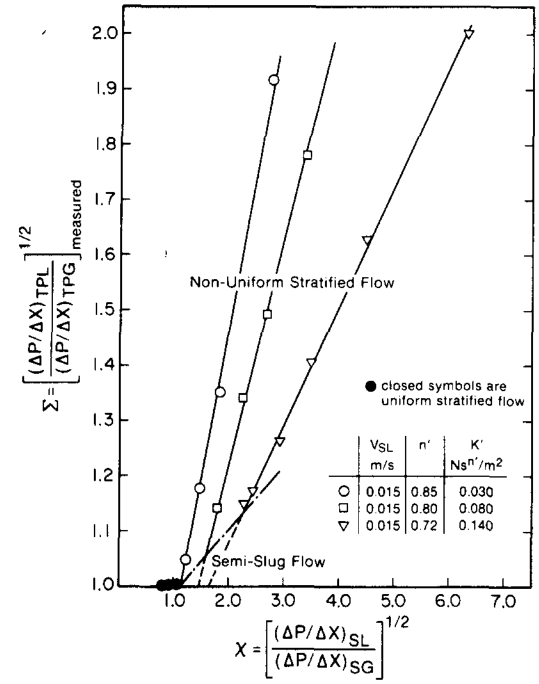

Figure 8. ILG parameter (Σ) vs Lockhart–Martinelli parameter χ for different polymer solutions.

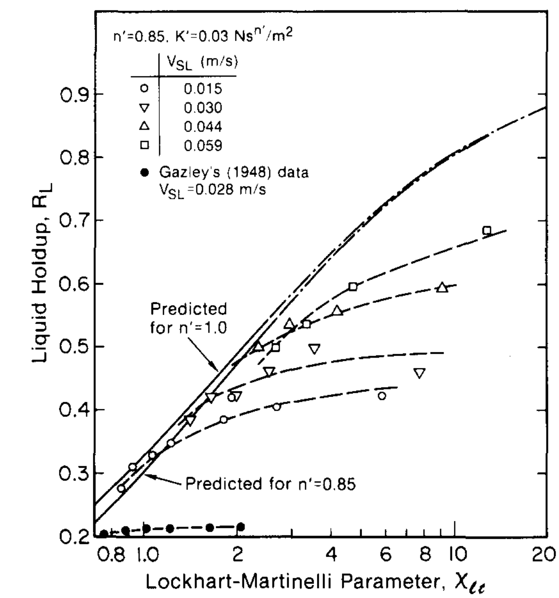

Figure 9. Comparison of liquid holdup data with values predicted by Heywood–Charles (1979) uniform stratified flow theory.

solution–air system are attributed primarily to the higher effective viscosity of the water-soluble polymer solution. It is difficult to determine from figure 9 if there is a significant difference in the rate of change of holdup with changes in the two-phase Lockhart–Martinelli parameter χ for the water–air (Gazley 1948) and polymer solution–air data.

The uniform stratified flow data points from figure 9 are plotted on an expanded scale in figure 10 in order to compare more critically the agreement with the Heywood–Charles (1979) model. For the uniform flow holdup data shown, there is an excellent agreement with the Heywood–Charles non-Newtonian liquid–gas stratified flow model. Not only do the uniform flow data fit a uniform flow model but the data fit the predicted curve for $n' = 0.85$ much better than they do for $n' = 1$; the maximum difference is $< 10\%$ and the average difference is $< 2\%$. Although $n' = 0.85$ is not highly non-Newtonian, the excellent agreement tends to validate the assumptions used in the model for the gas-phase equivalent diameter and the equality of τ_{iL}, τ_{iG} and τ_{WG}. Because of the shear-thinning nature of the liquid, $\tau_{iG} < \tau_{WG}$ might have been expected. Bishop & Deshpande (1986) found that for most laminar liquid–turbulent gas Newtonian uniform stratified data reported $\tau_{iG} \simeq \tau_{iL} \simeq \tau_{WG}$. The interfacial shear stress relationship was calculated using [4] and [5]. The results shown in figure 11 indicate that τ_{iG}/τ_{WG} is about 80% of τ_{iL}/τ_{WG} and is in the direction expected. It was also found that the predictions of holdup using [4] and [5] are not overly sensitive to the value of τ_{iL}/τ_{WG} used.

In their preliminary report Deshpande & Bishop (1983) used [10] and its equivalent [28] to calculate liquid holdup in non-uniform stratified flow. The experimental values of R_L and the values predicted by [28] are compared in figure 12. Although this procedure only checked the form of [28], the agreement is within $\pm 15\%$. The problem which arises in using [10] or [28] is that $f_{iL}/f_{iG} > 1$ and is not constant in non-uniform flow.

The ϕ_L^2 vs χ uniform stratified flow data shown in figure 13 are in good agreement with the Heywood–Charles (1979) model. Uniform stratified flow data should be obtained at lower values of the flow index n and for different geometries.

Data obtained at equal values of the flow index n' are compared in figure 14 for stratified flow through a 0.052 and a 0.025 m dia test section. At $\chi = 6.0$ and an *in situ* gas velocity

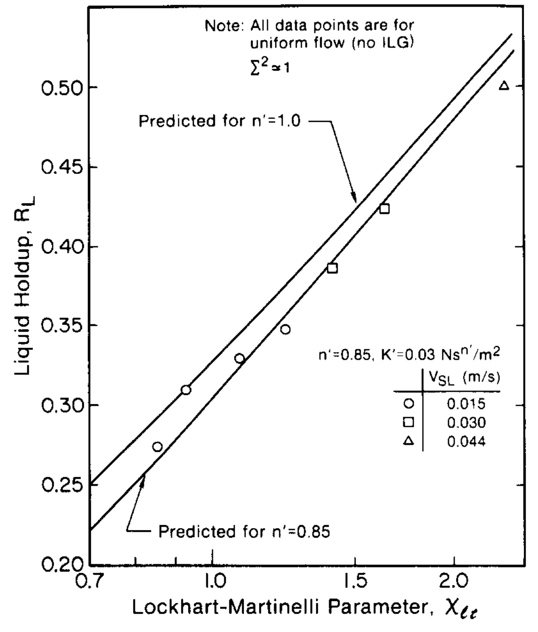

Figure 10. Experimental vs predicted liquid holdup in uniform stratified non-Newtonian liquid–gas flow.

equal to $V_G = 1$ m/s, the liquid holdup is larger and the non-uniformity ($\Sigma^2 = 2.5$ compared to $\Sigma^2 = 6.0$) is smaller in a smaller diameter tube. Thus although the liquid-phase resistance is larger in the smaller diameter tube, the higher interfacial gas-phase shear stress in the 0.025 m dia tube decreases the ILG, thereby providing greater flow uniformity in the smaller diameter tube. Very limited experimental data was collected in the 0.025 m dia pipe because of the difficulty in obtaining smooth stratified flow using high-viscosity liquids.

A comparison of the observed stratified flow-pattern data with the predictions of the Taitel–Dukler (1976b) map are shown in figure 15. All the data fall well within the predicted smooth stratified flow region. Because some data are obtained near the SS–WS and/or SS–I boundaries, these "boundary" data should lie closer to the predicted boundary (the Taitel–Dukler K parameter values observed differ from the predicted values by a factor of almost 100). However, Bishop & Deshpande (1986) suggest that the Taitel–Dukler SS–SW boundary is too high (by a factor of $\geqslant 10$). The overall results indicate that the sheltering coefficient used to determine the boundary ($S = 0.01$) should be greater than the value $S = 0.3$ suggested by Jeffreys (1925), assuming that the Jeffrey model is valid for flow in a closed channel.

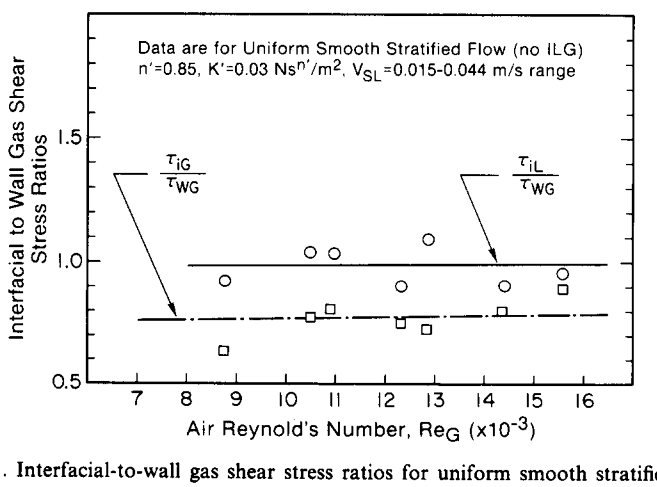

Figure 11. Interfacial-to-wall gas shear stress ratios for uniform smooth stratified flow.

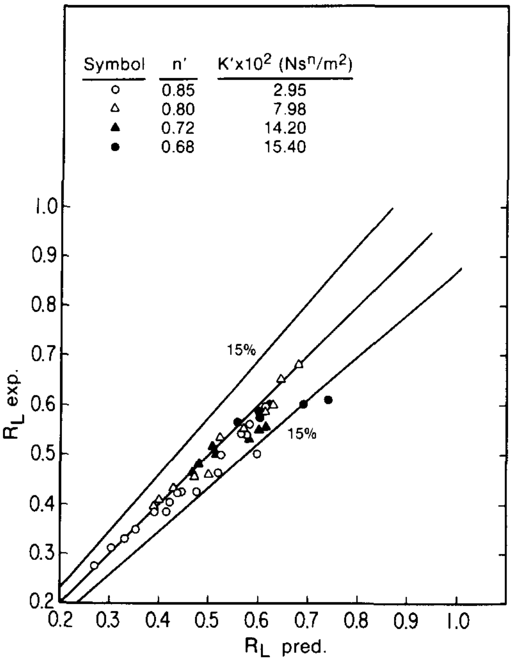

Figure 12. Comparison between experimental and predicted liquid holdup data.

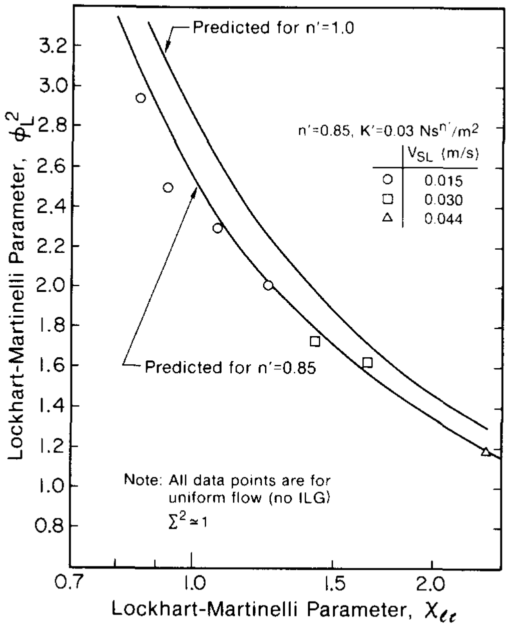

Figure 13. Predicted vs experimental two-phase pressure drop in uniform stratified non-Newtonian liquid–gas flow.

An interesting aspect of the interfacial gas-phase shear stress is shown in figure 16. The results were calculated using [1] and [2]. At a fixed value of the *in situ* gas velocity, the gas-phase shear stress decreases and appears to swing clockwise as the flow index parameter n decreases. This general behavior is consistent with the shear-thinning aspect of the non-Newtonian liquid used. The pivot point appears to occur at an *in situ* gas-phase Reynolds number equal to 6000. This value of the gas-phase Reynolds number coincides with what was assessed to be "fully" developed turbulent flow for the gas phase in these tests. Gazley (1948) also suggested that fully turbulent gas flow in his air–water system appeared to occur at $Re_G \approx 6000$. If lower values of the flow index had been tested, it might be speculated that at $n' = 0$, the gas-phase interfacial shear stress would be constant and independent of the air velocity. A justification for this idea is seen in the basic shear stress relation $\tau = K(\dot{\gamma})^n$, where at $n = 0$ the shear stress is constant and is numerically equal to the magnitude of the consistency index K. However, if the pivot point were to remain fixed at $Re_G = 6000$, a parallel line through this hub gives a value of $\tau_{iG} \approx 0.08$, whereas the expected value for K for $n' = 0$ would be > 0.15, the K value for the lowest n'-tested ($n' = 0.68$).

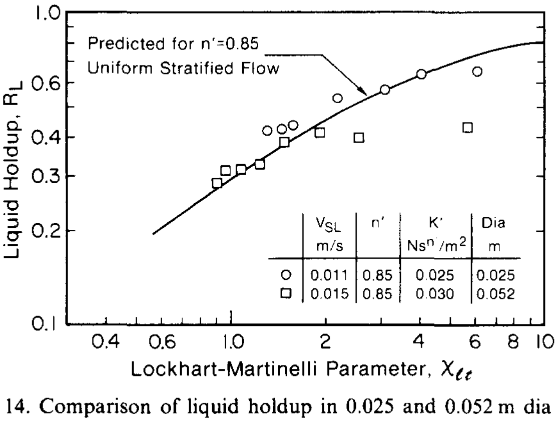

Figure 14. Comparison of liquid holdup in 0.025 and 0.052 m dia tubes.

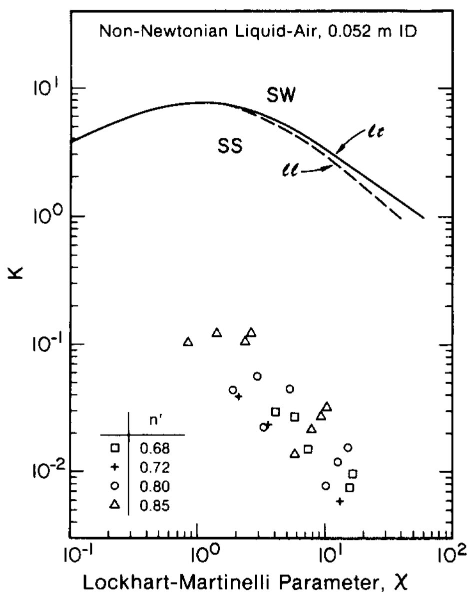

Figure 15. Comparison of observed smooth stratified flow data with the predictions of Taitel–Dukler (1976b) theory.

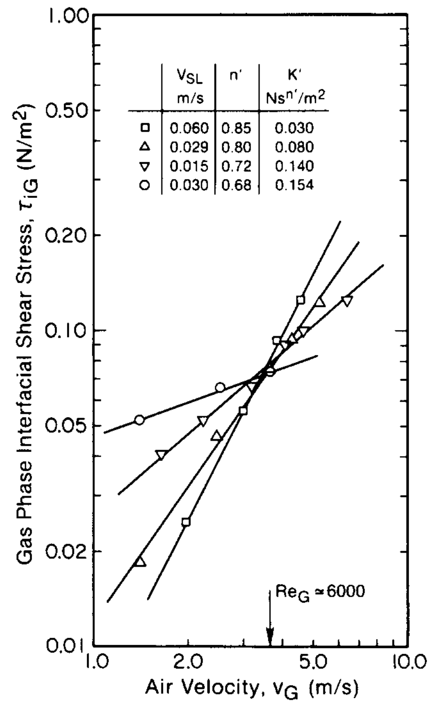

Figure 16. Gas-phase interfacial shear stress as a function of true gas velocity.

Mujawar & Rao (1981) indicated that stratified flow data were obtained using a solution of sodium alginate in water and air flowing through a 0.012 m dia tube. The liquid superficial velocities were reported to be 0.33 and 0.54 m/s and the air velocity was increased over a range of 0 to approx. 5 m/s. The value of the flow index was $n' = 0.94$, and $K' = 0.0058$ N $s^{n'}/m^2$; the liquid effective viscosity at the higher shear rate is 4.13 mPa s which is greater than that of water at the test temperature. The reported solution velocities are greater than the upper limit (≈ 0.2 m/s) of stratified flow given by current flow-pattern maps for water–air in 0.025 m dia tubes (Weisman *et al.* 1979; Taitel & Dukler 1976b). The experience of the current study is that uniform stratified flow (or any stratified flow pattern) could not be obtained at the liquid velocities cited. In fact, as discussed earlier, the liquid velocity has to be reduced to 0.015 m/s in order to obtain uniform stratified flow. This requirement of a low liquid velocity to achieve uniform smooth stratified flow for high-viscosity liquids is consistent with the results of Jensen (1972) who had to use liquid velocities as low as 7.5×10^{-4} m/s and an air velocity as high as 9 m/s to ensure uniform stratified flow through a 0.0375 m dia tube.

CONCLUSIONS

1. The Heywood–Charles flow model for predicting liquid holdup R_L and the two-phase drop ratio ϕ_L^2 is valid for uniform stratified flow of a pseudoplastic non-Newtonian liquid–gas mixture flowing through a cylindrical tube.
2. The Heywood–Charles model does not appear to be valid for predicting two-phase drag reduction in stratified flow because stratified flow transition to semi-slug flow occurred before the model criteria were reached.
3. Non-uniform stratified flow with an ILG will commonly occur in non-Newtonian liquid–gas systems (and in high-viscosity Newtonain liquid–gas systems) unless very low liquid velocities are used together with high gas velocities to suppress the interfacial gradient, thereby creating a horizontal interface.

4. The magnitude of non-uniformity is measured by the parameter Σ^2 which is the ratio of the measured two-phase pressure gradient in liquid phase to that in gas phase; indications are that $\Sigma^2 = 1$ (uniform flow) if $\chi^2 \approx 1$.
5. The assumption of $\tau_{iG}/\tau_{WG} = \tau_{iL}/\tau_{WG}$ is valid in non-Newtonian ($n' \approx 0.85$) laminar liquid–turbulent gas uniform stratified flow analysis. Moreover both ratios are approximately equal to unity with $\tau_{iG}/\tau_{WG} < \tau_{iL}/\tau_{WG}$.

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NOMENCLATURE

A = Area or shape factor
B = Shape factor
C_G = Constant in friction factor relationship for gas phase
D = Diameter
D_H = Hydraulic diameter
F = Function
f = Friction factor
Fr = Froude number
g = Acceleration due to gravity
h = Height
K = Flow-pattern parameter of Taitel & Dukler (1976a, b)
K' or K = Consistency index
L = Length
m = Exponent in friction factor relationship for gas phase
n or n' = Flow behavior index
P = Pressure
Q = Volumetric flow rate
q = Exponent in friction factor relationship for liquid phase
R = Holdup
Re = Reynolds number
S = Perimeter or sheltering coefficient
V = Velocity
W = Half the width of gas–liquid interface
x = Axial distance or length
Z = Parameter introduced in [7]

Subscripts

G = Gas
i = Interfacial
iG = Interfacial gas
iL = Interfacial liquid
L = Liquid
lt = Laminar–turbulent
m = Mean
MR = Metzner–Reed quantity
SG = Superficial gas
SL = Superficial liquid
TPG = Two-phase gas
TPL = Two-phase liquid

TPGM = Two-phase gas measured
TPLM = Two-phase liquid measured
tt = Turbulent–turbulent
WG = Wall gas
WL = Wall liquid

Greek symbols

χ = Lockhart–Martinelli parameter
μ = Viscosity
τ = Shear stress
α = Kinetic energy correction factor
ρ = Density
ϕ_L^2 = Two-phase pressure drop parameter
Σ^2 = Parameter defined in [9]
β = Tube inclination with horizontal
Δ = Indicates the difference
$\dot{\gamma}$ = Shear rate

Abbreviations

A = Annular
SS = Smooth stratified
SW = Stratified wavy
I = Intermittent
ILG = Interfacial level gradient

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